

# Semiclassical Analysis of M2-brane in $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$

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## Abstract

We start from the classical action describing a single M2-brane on  $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$  and consider semiclassical fluctuations around a static, 1/2 BPS configuration whose shape is  $\text{AdS}_2 \times \text{S}^1$ . The internal manifold  $\text{S}^7/\mathbb{Z}_k$  is described as a  $\text{U}(1)$  fibration over  $\text{CP}^3$  and the static configuration is wrapped on the  $\text{U}(1)$  fiber. Then the configuration is reduced to an  $\text{AdS}_2$  world-sheet of type IIA string on  $\text{AdS}_4 \times \text{CP}^3$  through the Kaluza-Klein reduction on  $\text{S}^1$ . It is shown that the fluctuations form an infinite set of  $\mathcal{N} = 1$  supermultiplets on  $\text{AdS}_2$  for  $k = 1, 2$ . The set is invariant under  $\text{SO}(8)$  which may be consistent with  $\mathcal{N} = 8$  supersymmetry on  $\text{AdS}_2$ . We discuss the behavior of the fluctuations around the boundary of  $\text{AdS}_2$  and its relation to deformations of Wilson loop operator.

# 1 Introduction

More than a decade has passed from the discovery of the AdS/CFT correspondence [1, 2]. It is still giving important arenas to study many aspects of string theory. Depending on the dimensionality, one can have various versions of AdS/CFT. Undoubtedly,  $\text{AdS}_5/\text{CFT}_4$  may be the most widely studied case. As for other cases, however, especially for  $\text{AdS}_4/\text{CFT}_3$ , there had been not so much progress because it seemed very difficult to construct a three-dimensional interacting  $\mathcal{N} = 8$  superconformal field theory which describes the low energy dynamics on multiple M2-branes [3].

A couple of years ago, one possible resolution for the problem of constructing such a field theory has been provided by Bagger, Lambert [4] and Gustavsson [5] (BLG) by utilizing the Lie 3-algebra. Although their works known as the BLG theory contain the unusual 3-algebra structure, the theory is in fact equivalent to the conventional gauge theory as shown in [6]. The BLG theory has triggered off a break-through in the study of  $\text{AdS}_4/\text{CFT}_3$ , and the equivalence of it with the conventional gauge theory has led Aharony, Bergman, Jafferis and Maldacena (ABJM) [7] to propose the duality between type IIA string theory on  $\text{AdS}_4 \times \mathbb{CP}^3$  and  $\mathcal{N} = 6$  superconformal Chern-Simons matter system in three dimensions. A numerous amount of the works related to this issue have been carried out so far, and hence our understanding on  $\text{AdS}_4/\text{CFT}_3$  is now making considerable progress.

By the way, an important check of the AdS/CFT duality is the correspondence between a  $1/2$  BPS Wilson loop and a string world-sheet whose shape is  $\text{AdS}_2$  [8, 9]. It is well studied in the case of  $\text{AdS}_5/\text{CFT}_4$  and it is an important observation that the exact expectation value of the  $1/2$  BPS circular Wilson loop can be computed by a Gaussian matrix model [10–12]. (For other works on supersymmetric Wilson loops, for example, see [13, 14].)

On the other hand, it is not so obvious to understand Wilson loops in  $\text{AdS}_4/\text{CFT}_3$ . There are  $1/2$  BPS  $\text{AdS}_2$  solutions in type IIA string theory on  $\text{AdS}_4 \times \mathbb{CP}^3$ . On the other hand, in the  $\mathcal{N} = 6$  Chern-Simons matter system, it is possible to construct at most  $1/6$  BPS Wilson loops [15–17] as far as the bosonic degrees of freedom are taken into account. It has been recently shown that  $1/2$  BPS Wilson loops can be constructed by including the fermionic degrees of freedom [18] (see [19] also). However, it is not clear why the fermionic contributions are necessary to realize the  $1/2$  BPS configuration while the corresponding  $1/2$  BPS string solution is purely bosonic. Another related question is what is the eleven-dimensional origin of the fermionic contributions to the  $1/2$  BPS Wilson loops.

Motivated by these questions, in this paper we will consider a semiclassical approximation

of a single M2-brane on  $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$  background<sup>1</sup>. For this purpose we begin with the classical action describing a single M2-brane on  $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$  and expand it around a static, 1/2 BPS classical solution. This solution<sup>2</sup> has the shape of  $\text{AdS}_2 \times \text{S}^1$ . The internal manifold  $\text{S}^7/\mathbb{Z}_k$  may be described as a  $\text{U}(1)$  fibration over  $\mathbb{CP}^3$  and the  $\text{S}^1$  part of the solution is wrapped on the  $\text{U}(1)$  fiber. Then the M2-brane solution is reduced to an  $\text{AdS}_2$  world-sheet of type IIA string on  $\text{AdS}_4 \times \mathbb{CP}^3$  through the Kaluza-Klein (KK) reduction on the  $\text{S}^1$ . We show that the fluctuations form an infinite set of  $\mathcal{N} = 1$  supermultiplets on  $\text{AdS}_2$  for  $k = 1, 2$  and that the set is composed of  $\mathcal{N} = 8$  supermultiplets on  $\text{AdS}_2$ . Finally we discuss what kind of the fluctuations can reach the boundary consistently.

This paper is organized as follows. In section 2, we introduce the classical action of a single M2-brane on  $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$ . Our notation and convention are also summarized. In section 3, we introduce a static, classical M2-brane solution whose shape is  $\text{AdS}_2 \times \text{S}^1$  and discuss a semiclassical approximation around the solution.<sup>3</sup> In section 4, we consider the KK reduction of the classical solution and the semiclassical fluctuations around it. The resulting spectrum consists of an infinite number of  $\mathcal{N} = 1$  supermultiplet on  $\text{AdS}_2$  for  $k = 1, 2$ . It is shown that the  $\mathcal{N} = 1$  supermultiplets can be combined to form an infinite set of the  $\mathcal{N} = 8$  supermultiplet on  $\text{AdS}_2$ . In section 5 we investigate the boundary behaviour of the fluctuations and identify the modes that can reach the boundary. The final section is devoted to conclusion and discussion.

## 2 M2-brane action on $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$

The starting point of our discussion is the classical action describing a single M2-brane on  $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$ . The purpose of this section is to introduce it and summarize the notation and convention utilized in this paper.

### 2.1 $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$ background

The  $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$  background is described by

$$ds_{11}^2 = \frac{R^2}{4} ds_{\text{AdS}_4}^2 + R^2 ds_{\text{S}^7/\mathbb{Z}_k}^2,$$

$$ds_{\text{AdS}_4}^2 = \frac{1}{z^2} (-dt^2 + dx_1^2 + dx_2^2 + dz^2),$$

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<sup>1</sup>For semiclassical strings on  $\text{AdS}_5 \times \text{S}^5$ , see for example the excellent reviews [20] and the references therein. Semiclassical approximation for Wilson loops was originally discussed in [21] and the quadratic action obtained there coincides with the non-relativistic action [22, 23]. The correspondence between the semiclassical approximation and the non-relativistic limit was confirmed also for the AdS-brane cases [24].

<sup>2</sup>The solution was originally argued in [8]. It is also discussed in classifying 1/2 BPS AdS-branes [25–27].

<sup>3</sup> The resulting action has the same form as the non-relativistic M2-brane action derived in [26].

$$ds_{S^7/\mathbb{Z}_k}^2 = ds_{\mathbb{CP}^3}^2 + \frac{1}{k^2}(dy + kA)^2, \quad (2.1)$$

with which the four-form field strength is equipped:

$$F_4 = -\frac{3}{8} \frac{R^3}{z^4} dt \wedge dx_1 \wedge dx_2 \wedge dz. \quad (2.2)$$

The radius of  $\text{AdS}_4$  is given by  $R_{\text{AdS}} = R/2$  while that of  $S^7$  is  $R$ . For notational convenience, we will set  $R = 1$  below. Note that the period of  $y$  is  $2\pi$ , i.e.,  $y \sim y + 2\pi$ .

The space  $S^7/\mathbb{Z}_k$  in (2.1) is described as a  $U(1)$  fibration over  $\mathbb{CP}^3$ . The field  $A$  is the one-form potential and leads to the Kähler form  $F \equiv \frac{1}{2}dA$  on  $\mathbb{CP}^3$ . For the explicit expression of the Kähler form  $F$ , we adopt the following one:

$$F = \frac{i}{2} \partial \bar{\partial} \ln(1 + w^m \bar{w}^m) = \frac{i}{2} \frac{dw^m \wedge d\bar{w}^n}{(1 + |w|^2)^2} \left( \delta_{mn}(1 + |w|^2) - \bar{w}^m w^n \right), \quad (2.3)$$

where  $w^m$  ( $m = 1, 2, 3$ ) are the complex coordinates on  $\mathbb{CP}^3$ . Then the metric on  $\mathbb{CP}^3$  is given by the Fubini-Study metric,

$$ds_{\mathbb{CP}^3}^2 = \frac{(1 + |w|^2)dw^n d\bar{w}^n - (\bar{w}^n w^m dw^n d\bar{w}^m)}{(1 + |w|^2)^2}. \quad (2.4)$$

From the metric (2.1), the elfbein  $e^A$  ( $A = 0, \dots, 9, \mathfrak{h}$ ) is taken as

$$e^a = \left( \frac{dt}{2z}, \frac{dx_1}{2z}, \frac{dx_2}{2z}, \frac{dz}{2z} \right), \quad e^{a'} = \left( e^{\tilde{a}}, e^{\mathfrak{h}} = \frac{1}{k}(dy + kA) \right), \quad (2.5)$$

where the index  $A$  is decomposed as  $A = (a, a') = (a, \tilde{a}, \mathfrak{h})$  with the following ranges:

- $a = 0, \dots, 3$  is the index for  $\text{AdS}_4$ ,
- $\tilde{a} = 4, \dots, 9$  is for  $\mathbb{CP}^3$ ,
- $a' = (\tilde{a}, \mathfrak{h})$  is for  $S^7/\mathbb{Z}_k$ , where  $\mathfrak{h}$  denotes the  $U(1)$  fiber direction.

With the elfbein, the non-vanishing components of spin connection  $w^{AB}$  are evaluated as

$$\begin{aligned} w^{03} &= -\frac{dt}{z}, & w^{13} &= -\frac{dx_1}{z}, & w^{23} &= -\frac{dx_2}{z}, \\ w^{\tilde{a}\tilde{b}} &= \hat{w}^{\tilde{a}\tilde{b}} - F^{\tilde{a}\tilde{b}} e^{\mathfrak{h}}, & w^{\mathfrak{h}\tilde{a}} &= F^{\tilde{a}}_{\mathfrak{h}} e^{\tilde{b}}, \end{aligned} \quad (2.6)$$

and  $\hat{w}^{\tilde{a}\tilde{b}}$  is the spin connection on  $\mathbb{CP}^3$ . Note here that  $F^{\tilde{a}\tilde{b}}$  can be taken to be of the form

$$F^{\tilde{a}\tilde{b}} = \begin{pmatrix} \varepsilon & & \\ & \varepsilon & \\ & & \varepsilon \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (2.7)$$

without loss of generality.

## 2.2 M2-brane action on $\text{AdS}_4 \times \mathbf{S}^7 / \mathbb{Z}_k$

Let us consider the classical action describing a single M2-brane on  $\text{AdS}_4 \times \mathbf{S}^7 / \mathbb{Z}_k$  (rather than multiple M2-branes).<sup>4</sup> Apart from the  $\mathbb{Z}_k$  quotient, the corresponding action has already been constructed in [29] and one may consult it in this subsection.

The classical action of a single M2-brane is composed of the Nambu-Goto (NG) action and the Wess-Zumino (WZ) term:

$$S_{\text{M2}} = S_{\text{NG}} + S_{\text{WZ}}. \quad (2.8)$$

Firstly, the NG action is given by

$$S_{\text{NG}} = T \int d^3 \xi \sqrt{-\det g}, \quad g_{ij} = \mathbf{L}_i^A \mathbf{L}_j^B \eta_{AB}, \quad (2.9)$$

where  $T$  is the M2-brane tension and  $\xi^i = (\tau, \sigma, \rho)$  are the world-volume coordinates. The induced metric  $g_{ij}$  is given in terms of the super elfbein

$$\mathbf{L}_i^A = \partial_i Z^{\hat{M}} \mathbf{L}_{\hat{M}}^A, \quad Z^{\hat{M}} = (X^M, \theta) \quad (M = 0, \dots, 9, \mathfrak{t}). \quad (2.10)$$

In our notation of the eleven-dimensional superspace coordinate,  $Z^{\hat{M}}$ ,  $M$  labels the curved space-time coordinates, and  $\theta$  is the 32-component Majorana spinor. The indices  $A, B, \dots$  labels the local Lorentz frame.

For our purpose, it is sufficient to consider the super elfbein  $\mathbf{L}^A$  expanded explicitly up to the second order of  $\theta$ :

$$\mathbf{L}^A = e^A - \bar{\theta} \Gamma^A D\theta + O(\theta^4). \quad (2.11)$$

Here  $\Gamma^A$ 's are the  $\text{SO}(1,10)$  gamma matrices and  $\bar{\theta} = \theta^T C$  with the charge conjugation matrix  $C$ . The covariant derivative for  $\theta$  is defined as

$$D\theta \equiv d\theta - e^a I \Gamma_a \theta - \frac{1}{2} e^{a'} I \Gamma_{a'} \theta + \frac{1}{4} w^{AB} \Gamma_{AB} \theta, \quad I \equiv \Gamma^{0123}, \quad (2.12)$$

where the second and third terms come from the coupling to the four-form field strength  $F_4$ .

Another part of the M2-brane action, the WZ term, is given by

$$S_{\text{WZ}} = T \int \left[ {}^* c_{(3)} + \int_0^1 dt {}^* (-\hat{\mathbf{L}}^A \wedge \hat{\mathbf{L}}^B \wedge \hat{L} \Gamma_{AB} \theta) \right], \quad (2.13)$$

$$dc_{(3)} = -\frac{6}{4!} \epsilon_{a_1 \dots a_4} e^{a_1} \wedge \dots \wedge e^{a_4}, \quad (2.14)$$

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<sup>4</sup>A similar analysis for the finite values of  $k$  is discussed in [28], where spinning string solutions are considered in comparison to our static configuration.

where  $\hat{\mathbf{L}} = \mathbf{L}|_{\theta \rightarrow t\theta}$ ,  $\hat{L} = L|_{\theta \rightarrow t\theta}$  and we assume that  $L$  is defined up to the second order of  $\theta$ :

$$L = D\theta + O(\theta^3).$$

The  $*$ -operation means the pullback of the background geometry to the world-volume of M2-brane.

Let us now expand the M2-brane action up to the quadratic order in  $\theta$ . Since the background of our concern does not have non-vanishing fermionic field, there are no terms linear in  $\theta$ , and thus the expansion results in

$$S_{\text{M2}} = S_B + S_F + O(\theta^4), \quad (2.15)$$

where  $S_B$  is the purely bosonic part, that is, the zeroth order part in  $\theta$ , and  $S_F$  is the part of quadratic order. Before presenting each part of the action, it is convenient to expand the metric in powers of  $\theta$  as<sup>5</sup>

$$g_{ij} = g_{Bij} + g_{Fij} + O(\theta^4), \quad (2.16)$$

$$g_{Bij} = e_i^A e_j^B \eta_{AB}, \quad (2.17)$$

$$g_{Fij} = -2e_i^A \bar{\theta} \Gamma^B D_j \theta \eta_{AB}, \quad (2.18)$$

where  $e_i^A \equiv \partial_i X^M e_M^A$ . Then the purely bosonic part is written down with  $g_{Bij}$  and  $c_{(3)}$  as

$$S_B = T \int d^3\xi \sqrt{-\det g_B} + T \int *c_{(3)}, \quad (2.19)$$

where, from Eqs. (2.5) and (2.14), we may take

$$c_{(3)} = -\frac{1}{8z^3} dt \wedge dx_1 \wedge dx_2. \quad (2.20)$$

The quadratic part  $S_F$  is given by

$$S_F = \frac{T}{2} \int d^3\xi \sqrt{-\det g_B} g_B^{ij} g_{Fij} + \frac{T}{2} \int d^3\xi \epsilon^{ijk} e_i^A e_j^B \bar{\theta} \Gamma_{AB} D_k \theta. \quad (2.21)$$

This seems to have a bit unusual form. Although one may proceed with it, it is useful to rewrite the action so that the structure is manifest and understandable as much as possible. In order to rewrite the action, let us define the following quantities:

$$\Gamma_i \equiv e_i^A \Gamma_A, \quad \tilde{\Gamma} \equiv \frac{1}{3!} \epsilon^{ijk} \Gamma_{ijk}. \quad (2.22)$$

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<sup>5</sup> We use the following notation:  $a_{(i}b_{j)} = \frac{1}{2}(a_i b_j + a_j b_i)$  and  $a_{[i}b_{j]} = \frac{1}{2}(a_i b_j - a_j b_i)$ .

Then it is an easy task to check that

$$\frac{1}{2}\epsilon^{ijk}\Gamma_{ij} = \Gamma^k\tilde{\Gamma}, \quad \Gamma^i = g_B^{ij}\Gamma_j. \quad (2.23)$$

By exploiting these formulae related to  $\Gamma_i$  and introducing another quantity

$$\Gamma \equiv \frac{\tilde{\Gamma}}{\sqrt{-\det g_B}}, \quad (2.24)$$

the action  $S_F$  can be rewritten as

$$S_F = -T \int d^3\xi \sqrt{-\det g_B} \bar{\theta} \Gamma^i (1 - \Gamma) D_i \theta. \quad (2.25)$$

This is a form with the clearer structure as desired.

Before closing this section, as one important remark, let us notice the presence of  $1 - \Gamma$  in the action. As particular properties of  $\Gamma$ , one can show that  $\Gamma^2 = 1$  with  $\tilde{\Gamma}^2 = -\det g_B$  and  $\text{tr} \Gamma = 0$ . This implies that half of the 32 components of  $\theta$  are redundant in the action and decoupled from the other dynamical variables. Indeed, this is nothing but the consequence of the  $\kappa$  symmetry that the M2-brane action possesses.

### 3 Semiclassical analysis of an $\text{AdS}_2 \times \text{S}^1$ -brane

We consider a classical configuration of a single M2-brane embedded in  $\text{AdS}_4 \times \text{S}^7 / \mathbb{Z}_k$ . It may be considered as the dual to a Wilson line of the boundary superconformal field theory. We study the quadratic action describing the fluctuations around it.

#### 3.1 Static Classical Solution

Let us consider the static configuration,

$$t = \tau, \quad z = \sigma, \quad y = \rho, \quad x_1 = x_2 = w^m = 0, \quad (3.1)$$

$$\theta = 0. \quad (3.2)$$

This configuration satisfies the equation of motion for a single M2-brane and preserves half the supersymmetries of the background geometry. Note that the equation of motion itself is the one derived from the purely bosonic action (2.19) because  $\theta = 0$  for the above configuration. The world-volume of the M2-brane described by the configuration touches the  $\text{AdS}_4$  boundary on which the three-dimensional superconformal field theory lives.

When remembering the case of  $\text{AdS}_5/\text{CFT}_4$ , the boundary of this kind of static configuration represents a Wilson line. Similarly, it would be natural to regard the boundary of our

configuration as a Wilson line in the boundary theory. In fact, the configuration described by (3.1) and (3.2) is reduced to a 1/2 BPS string world-sheet in type IIA string theory through the dimensional reduction. Then it is argued that the resulting configuration would correspond to a Wilson loop in the context of  $\text{AdS}_4/\text{CFT}_3$  based on the ABJM model. In other words, the configuration given by (3.1) and (3.2) can be regarded as the up-lift of the type IIA string world-sheet to M-theory. According to this up-lift, the Wilson loop gets corrections by KK modes and hence there might be a possibility that it cannot be understood as a Wilson loop any more. However, it is supersymmetric and so it possibly remains understandable as the Wilson loop even in the M-theory limit.

Next we shall consider fluctuations around the configuration given by (3.1) and (3.2). We will deal with the bosonic and fermionic fluctuations separately in the following two subsections.

### 3.2 Bosonic fluctuations

Assuming that the bosonic fluctuations are transverse to the static configuration (3.1), we consider the following expansion of the fields:

$$t = \tau, \quad x_1 = 0 + \tilde{x}_1, \quad x_2 = 0 + \tilde{x}_2, \quad z = \sigma, \quad w^m = 0 + \zeta^m, \quad y = \rho. \quad (3.3)$$

Here the fields,  $\tilde{x}_1$ ,  $\tilde{x}_2$ ,  $\zeta$ , and  $\bar{\zeta}$ , denote the bosonic fluctuations. Because the classical configuration (3.1) plays the role in fixing the world-volume diffeomorphism, there are no fluctuations along the world-volume.

Then we obtain that

$$S_B = S_B^{(0)} + S_B^{(1)} + S_B^{(2)} + \dots, \quad (3.4)$$

where  $S_B^{(n)}$  is the part of the action containing the terms of the  $n$ -th order of the bosonic fluctuations. Note that  $S_B^{(1)}$  vanishes due to the equations of motion. The zeroth-order part is just the action for the classical configuration and is computed as

$$S_B^{(0)} = T \int d^3\xi \frac{1}{4k\sigma^2}.$$

Although the value of this action leads to the divergent contribution proportional to the volume, it can be eliminated by the Legendre transformation as discussed in [30] even for the M2-brane case.

Before proceeding further, let us see the absence of such divergence by following the prescription of [30]. We begin with a variation of the bosonic M2-brane action,  $S_B$  of (2.19), which is obtained as

$$\delta S_B = T \int d^3\xi \partial_\sigma (\delta X^M P_M^\sigma) = T \int d\tau d\rho (\delta X^M P_M^\sigma) \Big|_{\sigma=0}. \quad (3.5)$$



The total derivative terms with respect to  $\tau$  and  $\rho$  vanish while the one with respect to  $\sigma$  remains since the string world-sheet has the boundary at  $\sigma = 0$ . In this variation we have used the equation of motion

$$\frac{\partial \mathcal{L}}{\partial X^M} - \partial_i \left( \frac{\partial \mathcal{L}}{\partial (\partial_i X^M)} \right) = 0 \quad (3.6)$$

and defined  $P_M^\sigma$  by

$$P_M^\sigma \equiv \frac{\partial \mathcal{L}}{\partial (\partial_\sigma X^M)} . \quad (3.7)$$

The result implies that the action  $S_B$  is a function of  $X^M$  on the boundary. By the way, the Wilson line is a function of  $X^\mu = (t, x_1, x_2)$ , not of  $Y^{\mu'} = (z, w^m, y)$ . In order to make a connection with the Wilson line, we consider the Legendre transformation  $S' = S_B + S_L$  with

$$S_L = -T \int d\tau d\rho Y^{\mu'} P_{\mu'}^\sigma . \quad (3.8)$$

The variation of the transformed action is obtained as

$$\delta S' = T \int d\tau d\rho (\delta X^\mu P_\mu^\sigma + Y^{\mu'} \delta P_{\mu'}^\sigma) \Big|_{\sigma=0} . \quad (3.9)$$

This means that the action  $S'$  is a function of  $X^\mu$  and  $P_{\mu'}^\sigma$  and thus we see that  $S'$  is more appropriate to examine the correspondence to the Wilson line.

Having the suitable action  $S'$ , we are ready to investigate the divergence structure, which is basically given by evaluating  $S_B$  and  $S_L$ . The value of  $S_L$  is evaluated in our static gauge as

$$S_L = -T \int d\tau d\rho z \frac{\partial_\sigma z}{4kz^2} \Big|_{\sigma=\epsilon} = -T \int d\tau d\rho \frac{1}{4k\epsilon} , \quad (3.10)$$

where taking the limit of  $\epsilon \rightarrow 0$  is assumed implicitly. On the other hand, the divergent contribution from  $S_B^{(0)}$  is evaluated as

$$S_B^{(0)} = T \int d^3\xi \frac{1}{4k\sigma^2} = T \int d\tau d\rho \left( -\frac{1}{4k\sigma} \right) \Big|_{\sigma=\epsilon}^{\sigma=\infty} = +T \int d\tau d\rho \frac{1}{4k\epsilon} . \quad (3.11)$$

Thus the divergence from  $S_B^{(0)}$  and  $S_L$  cancels out each other, and the volume-divergence in  $S_B^{(0)}$  can be eliminated.

Now let us return to our main concern, that is,  $S_B^{(2)}$ . In order to extract  $S_B^{(2)}$ , we first note that the Kähler form  $F$  is expanded as

$$F = \frac{i}{2} d\zeta \wedge d\bar{\zeta} + \cdots , \quad (3.12)$$

and the one-form potential  $A$  becomes

$$A = \frac{i}{2}(\zeta d\bar{\zeta} - \bar{\zeta} d\zeta) + \dots,$$

where “...” represents the terms with higher-order in fluctuations (higher than the quadratic order). Then the metric  $g_{Bij}$  is expanded as follows:

$$\begin{aligned} g_{Bij} &= g_{Bij}^{(0)} + g_{Bij}^{(2)} + \dots, \\ g_{Bij}^{(0)} &= \text{diag} \left( -\frac{1}{4\sigma^2}, \frac{1}{4\sigma^2}, \frac{1}{k^2} \right), \\ g_{Bij}^{(2)} &= \frac{1}{4\sigma^2} \partial_{(i} \chi \partial_{j)} \bar{\chi} + \partial_{(i} \zeta^m \partial_{j)} \bar{\zeta}^m + \frac{i}{k} \partial_{(i} \rho (\zeta \partial_{j)} \bar{\zeta} - \bar{\zeta} \partial_{j)} \zeta), \end{aligned} \quad (3.13)$$

where  $\tilde{x}_1$  and  $\tilde{x}_2$  have been combined to form a complex field  $\chi$ ,

$$\chi = \tilde{x}_1 + i\tilde{x}_2.$$

The expanded metric subsequently allows us to expand the determinant of the metric as

$$\begin{aligned} \sqrt{-\det g_B} &= \sqrt{-\det g_B^{(0)}} \sqrt{\det(1 + (g_B^{(0)})^{-1} g_B^{(2)} + \dots)} \\ &= \sqrt{-\det g_B^{(0)}} \left( 1 + \frac{1}{2} \text{tr}((g_B^{(0)})^{-1} g_B^{(2)}) \right) + \dots. \end{aligned}$$

We are now ready to write down the quadratic action  $S_B^{(2)}$ . For the NG action, we find that the quadratic part is given by

$$(S_{\text{NG}})_B^{(2)} = TR^3 \int d^3\xi \sqrt{-\det g_B^{(0)}} \left[ \frac{1}{2} g_B^{(0)ij} \left( \frac{1}{4\sigma^2} \partial_i \chi \partial_j \bar{\chi} + \partial_i \zeta^m \partial_j \bar{\zeta}^m \right) + \frac{i}{2} k (\zeta \partial_\rho \bar{\zeta} - \bar{\zeta} \partial_\rho \zeta) \right].$$

The contribution from the WZ term becomes

$$(S_{\text{WZ}})_B^{(2)} = -TR^3 \int d^3\xi \frac{3i}{32\sigma^4} (\chi \partial_\rho \bar{\chi} - \bar{\chi} \partial_\rho \chi), \quad (3.14)$$

with the help of integration by parts.

At this point, there is a comment on the overall factor  $TR^3$ . It can be absorbed into the fluctuations by the redefinition,

$$\sqrt{TR^3} \chi \rightarrow \chi, \quad \sqrt{TR^3} \zeta \rightarrow \zeta, \quad (3.15)$$

and disappears from the action. As for the higher-order contributions  $S^{(n>2)}$ , however, we have inverse powers of the factor after the redefinition. Thus, by take the limit  $TR^3 \rightarrow \infty$ , all such contributions simply vanish and what remains is the contributions up to the quadratic order.

After the redefinition (3.15), the quadratic action for the bosonic part is obtained as

$$\begin{aligned}
S_B^{(2)} &= (S_{\text{NG}})_B^{(2)} + (S_{\text{WZ}})_B^{(2)} \\
&= \int d^3\xi \sqrt{-\det g_B^{(0)}} \left[ \frac{1}{2} g_B^{(0)ij} \left( \frac{1}{4\sigma^2} \partial_i \chi \partial_j \bar{\chi} + \partial_i \zeta^m \partial_j \bar{\zeta}^m \right) \right. \\
&\quad \left. + \frac{i}{2} k (\zeta \partial_\rho \bar{\zeta} - \bar{\zeta} \partial_\rho \zeta) - \frac{3i}{8} k \frac{1}{\sigma^2} (\chi \partial_\rho \bar{\chi} - \bar{\chi} \partial_\rho \chi) \right]. \quad (3.16)
\end{aligned}$$

By introducing a new field  $\eta$  defined as

$$\eta \equiv \frac{1}{2\sigma} \chi, \quad (3.17)$$

we obtain the canonical action given by

$$\begin{aligned}
S_B^{(2)} &= \frac{1}{2} \int d^3\xi \sqrt{-\det g_B^{(0)}} \left[ g_B^{(0)ij} (\partial_i \eta \partial_j \bar{\eta} + \partial_i \zeta^m \partial_j \bar{\zeta}^m) + 8\eta \bar{\eta} \right. \\
&\quad \left. + ik(\zeta \partial_\rho \bar{\zeta} - \bar{\zeta} \partial_\rho \zeta) - 3ik(\eta \partial_\rho \bar{\eta} - \bar{\eta} \partial_\rho \eta) \right]. \quad (3.18)
\end{aligned}$$

The relation (3.17) will be important also when we discuss its behavior near the boundary.

### 3.3 Fermionic fluctuations

Next we consider the fermionic fluctuations around the solution (3.1) and (3.2). Since the classical value of  $\theta$  is zero, the action for the fermionic fluctuations is obtained simply by substituting (3.1) into the action in  $S_F$  (2.25) and regarding  $\theta$  as the fluctuation.

For notational clarity, we first put a bar to the ingredients of  $S_F$  evaluated at the classical solution (3.1), namely

$$\begin{aligned}
\bar{\Gamma}_i &= \left( \frac{1}{2\sigma} \Gamma_0, \frac{1}{2\sigma} \Gamma_3, \frac{1}{k} \Gamma_{\natural} \right), \quad \bar{\Gamma}^i = (-2\sigma \Gamma_0, 2\sigma \Gamma_3, k \Gamma_{\natural}) , \\
\bar{w}_\tau^{03} &= -\frac{1}{\sigma}, \quad \bar{w}^{13} = \bar{w}^{23} = 0, \quad \bar{w}_\rho^{\bar{a}\bar{b}} = -F^{\bar{a}\bar{b}} \frac{1}{k}, \quad \bar{w}^{\natural\bar{a}} = 0. \quad (3.19)
\end{aligned}$$

Then the covariant derivative becomes

$$\begin{aligned}
\bar{D}_\tau \theta &= \partial_\tau \theta - \frac{1}{2\sigma} \Gamma_{03} \theta + \frac{1}{2\sigma} \Gamma^{123} \theta, \\
\bar{D}_\sigma \theta &= \partial_\sigma \theta - \frac{1}{2\sigma} \Gamma^{012} \theta, \\
\bar{D}_\rho \theta &= \partial_\rho \theta - \frac{1}{2k} \Gamma^{0123}{}_{\natural} \theta - \frac{1}{4k} F^{\bar{a}\bar{b}} \Gamma_{\bar{a}\bar{b}} \theta. \quad (3.20)
\end{aligned}$$

Because the action  $S_F$  in (2.25) is already quadratic in  $\theta$ , the action evaluated at the classical solution (3.1) is the desired one, that is,  $S_F^{(2)}$ . As briefly mentioned just after (2.25), the M2-brane action has the fermionic  $\kappa$  symmetry, which should be fixed before doing any practical calculation with the action. Here, from the  $\kappa$  symmetry transformation rule for  $\theta$ ,

$$\delta_\kappa \theta = (1 + \Gamma)\kappa$$

and the form of  $S_F$ , we simply take the  $\kappa$  symmetry fixing condition as

$$(1 + \Gamma)\theta = 0, \quad (3.21)$$

which is used usually when one considers the number of supersymmetries preserved by a brane configuration in a given supersymmetric background. Then, with the fixing condition (3.21), the quadratic action for the fermionic part is obtained as

$$S_F^{(2)} = -2 \int d^3\xi \sqrt{-\det g_B^{(0)}} \left[ \bar{\theta}_- \bar{\Gamma}^\tau (\partial_\tau - \frac{1}{2\sigma} \Gamma_{03}) \theta_- + \bar{\theta}_- \bar{\Gamma}^\sigma \partial_\sigma \theta_- + \bar{\theta}_- \bar{\Gamma}^\rho \partial_\rho \theta_- \right. \\ \left. + \frac{3}{2} \bar{\theta}_- \Gamma^{0123} \theta_- - \frac{1}{2} \bar{\theta}_- \Gamma_{\mathfrak{q}} (\Gamma_{45} + \Gamma_{67} + \Gamma_{89}) \theta_- \right], \quad (3.22)$$

where we have introduced  $\theta_-$  defined as

$$\theta_- \equiv P_- \theta, \quad P_- = \frac{1}{2} (1 - \Gamma_{03\mathfrak{q}}), \quad (3.23)$$

and the overall factor  $TR^3$  has been absorbed into  $\theta_-$  by the redefinition  $\sqrt{TR^3} \theta_- \rightarrow \theta_-$ . After the redefinition as in the bosonic case, the higher-order contributions in fluctuations vanish in the limit  $TR^3 \rightarrow \infty$ .

Since we are studying the fluctuations on the three-dimensional world-volume of M2-brane, it is convenient to rewrite this action in such a way that makes the three-dimensional structure manifest. What we should do first is to take a suitable representation of the gamma matrices with the manifest  $SO(1,2) \times SO(8)$  structure. A possible representation is the following,

$$\begin{aligned} \Gamma_0 &= \rho_0 \otimes \gamma_9, & \gamma_9 &= \gamma_{1\dots 8} = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3, \\ \Gamma_3 &= \rho_1 \otimes \gamma_9, & \rho_\alpha &= (i\sigma_2, \sigma_1, \sigma_3), \\ \Gamma_{\mathfrak{q}} &= \rho_2 \otimes \gamma_9, & & \\ \Gamma_1 &= 1 \otimes \gamma_1, & \gamma_1 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3, \\ \Gamma_2 &= 1 \otimes \gamma_2, & \gamma_2 &= \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3, \\ \Gamma_4 &= 1 \otimes \gamma_3, & \gamma_3 &= 1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_3, \\ \Gamma_5 &= 1 \otimes \gamma_4, & \gamma_4 &= 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3, \\ \Gamma_6 &= 1 \otimes \gamma_5, & \gamma_5 &= 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3, \end{aligned}$$

$$\begin{aligned}
\Gamma_7 &= 1 \otimes \gamma_6 , & \gamma_6 &= 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 , \\
\Gamma_8 &= 1 \otimes \gamma_7 , & \gamma_7 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 , \\
\Gamma_9 &= 1 \otimes \gamma_8 , & \gamma_8 &= 1 \otimes 1 \otimes 1 \otimes \sigma_2 , 
\end{aligned} \tag{3.24}$$

where  $\rho_\alpha$  are the gamma matrices in three dimensions, and  $\{\gamma_1, \dots, \gamma_8\}$  are the  $SO(8)$  gamma matrices. In this representation, the  $B$ -conjugation matrix  $B$  is defined as

$$\Gamma_A^* = +B\Gamma_A B^{-1},$$

and satisfy the relations  $B^\dagger B = 1$  and  $B^T = B$ . The matrix  $B$  may be represented by

$$B = \Gamma_{2579} = 1 \otimes i\sigma_2 \otimes \sigma_1 \otimes i\sigma_2 \otimes \sigma_1. \tag{3.25}$$

Then the charge conjugation matrix  $C = BA^\dagger$  with  $A = \Gamma_0$  is given by

$$C = -\Gamma_{02579} = \rho_0 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_2. \tag{3.26}$$

By using this representation, the action reduces to

$$\begin{aligned}
S_F^{(2)} &= -2 \int d^3\xi \sqrt{-\det g_B^{(0)}} \left[ \theta_-^T \rho_0 \rho^i \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_2 \nabla_i \theta_- \right. \\
&\quad \left. - \frac{i}{2} \theta_-^T \rho_0 \rho_2 \otimes \left( 3\sigma_1 \sigma_3 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_2 + \sigma_1 \otimes \sigma_2 \sigma_3 \otimes \sigma_1 \otimes \sigma_2 \right. \right. \\
&\quad \left. \left. + \sigma_1 \otimes \sigma_2 \otimes \sigma_1 \sigma_3 \otimes \sigma_2 + \sigma_1 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_2 \sigma_3 \right) \theta_- \right]. 
\end{aligned} \tag{3.27}$$

Here we have introduced the spinor covariant derivative  $\nabla_i$  on the world-volume as follows. From  $g_{Bij}^{(0)}$  in (3.13), the dreibein and the spin connection are obtained as

$$\hat{e}^\alpha = \left( \frac{d\tau}{2\sigma}, \frac{d\sigma}{2\sigma}, \frac{d\rho}{k} \right), \quad \hat{w}^0{}_1 = -\frac{d\tau}{\sigma}. \tag{3.28}$$

Then the spinor covariant derivative on the world-volume  $\text{AdS}_2 \times \text{S}^1$  is given by

$$\begin{aligned}
\nabla_i &= \partial_i + \frac{1}{4} \hat{w}_i^{\alpha\beta} \rho_{\alpha\beta} = \left( \partial_\tau - \frac{1}{2\sigma} \rho_2, \partial_\sigma, \partial_\rho \right), \\
\rho_i &= \hat{e}^\alpha \rho_\alpha, \quad \rho^i = g_B^{(0)ij} \rho_j = (-2\sigma \rho_0, 2\sigma \rho_1, k \rho_2). 
\end{aligned} \tag{3.29}$$

In order to describe the spinors from the M2-brane world-volume perspective, we decompose the 32-component spinor  $\theta$  into 16 two-component complex spinors  $\vartheta^{\alpha_1 \dots \alpha_4}$  ( $\alpha_i = \pm$ ) where the four index  $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$  represent the four  $U(1)$  charges of  $U(1)^4 \subset SO(8)$ . We will see soon that the Majorana condition relates a half of complex spinors to another half of them by complex conjugation. Since the projector is represented by

$$P_- = 1 \otimes \frac{1}{2}(1 - \gamma_9) = 1 \otimes \frac{1}{2}(1_{16} - \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3), \tag{3.30}$$

$\theta_-$  is composed of 8 two-component complex spinors  $\vartheta^{\alpha_1 \dots \alpha_4}$  with  $\alpha_1 \alpha_2 \alpha_3 \alpha_4 = -1$ , namely

$$\vartheta^{-+++}, \quad \vartheta^{+++-}, \quad \vartheta^{+-++}, \quad \vartheta^{+--+}, \quad (3.31)$$

$$\vartheta^{+---}, \quad \vartheta^{----+}, \quad \vartheta^{--+-}, \quad \vartheta^{-+--}. \quad (3.32)$$

In terms of the two-component spinors, the action is rewritten as

$$S_F^{(2)} = -2 \int d^3 \xi \sqrt{-\det g_B^{(0)}} \sum_{\alpha_1 \alpha_2 \alpha_3 \alpha_4 = -1} \left[ -\alpha_2 \alpha_4 (\vartheta^{-\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4})^T \rho_0 \rho^i \nabla_i \vartheta^{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \right. \\ \left. + \frac{i}{2} \alpha_2 \alpha_4 (3\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) (\vartheta^{-\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4})^T \rho_0 \rho_2 \vartheta^{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \right], \quad (3.33)$$

where the summation is taken over the eight spinors in (3.31) and (3.32). To derive this expression, we have used  $\sigma_3 \vartheta^\alpha = \alpha \vartheta^\alpha$ ,  $\sigma_1 \vartheta^\alpha = \vartheta^{-\alpha}$  and  $\sigma_2 \vartheta^\alpha = -i \alpha \vartheta^{-\alpha}$ .

One may think that there is a problem at this point. While  $\theta_-$  has 16 independent real components, the naive counting of real components for the eight spinors  $\vartheta^{\alpha_1 \dots \alpha_4}$  gives 32. Actually, this point is not problematic and cured by the Majorana condition, which relates the four spinors in (3.31) to the four spinors in (3.32). The proof is as follows. The charge conjugate of a spinor  $\psi$  is given by

$$\psi^c = B^{-1} \psi^*, \quad (3.34)$$

where  $B$  is given in (3.25). The Majorana condition is the equality  $\psi^c = \psi$ , and thus the eleven dimensional Majorana spinor  $\theta_+$  satisfies

$$\theta_+ \equiv B^{-1} \theta_+^*.$$

This implies immediately that

$$\vartheta^{\alpha_1 \dots \alpha_4} = \alpha_1 \alpha_3 (\vartheta^{-\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4})^*. \quad (3.35)$$

Thus the independent spinors are provided by (3.31) or (3.32). Hereafter, we will take the spinors in (3.31) as the independent ones.

In fact, using the relations

$$\alpha_2 \alpha_4 (\vartheta^{-\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4})^T \rho_0 = \alpha_1 \alpha_2 \alpha_3 \alpha_4 (\vartheta^{\alpha_1 \alpha_2 \alpha_3 \alpha_4})^\dagger \rho_0 = -(\vartheta^{\alpha_1 \alpha_2 \alpha_3 \alpha_4})^\dagger \rho_0,$$

we obtain the following action

$$S_F^{(2)} = 4 \int d^3 \xi \sqrt{-\det g_B^{(0)}} \sum_{(3.31)} \left[ \bar{\vartheta}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \rho^i \nabla_i \vartheta^{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \right. \\ \left. - \frac{i}{2} (3\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) \bar{\vartheta}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \rho_2 \vartheta^{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \right], \quad (3.36)$$

where the bar denotes the Dirac conjugate:  $\bar{\vartheta} \equiv \vartheta^\dagger \rho_0^\dagger$ . This is obviously of the three-dimensional form and  $\vartheta^{\alpha_1 \dots \alpha_4}$  are massive complex fermions propagating on  $\text{AdS}_2 \times \text{S}^1$  with the metric  $g_{Bij}^{(0)}$  of (3.13).

## 4 KK Reduction from $\text{AdS}_2 \times \mathbb{S}^1$ to $\text{AdS}_2$

The M2-brane considered in the previous section is described by the three-dimensional field theory on the  $\text{AdS}_2 \times \mathbb{S}^1$  world-volume. We now take  $\mathbb{S}^1$  as the M-theory circle, and perform the KK reduction. Then the resulting two-dimensional theory is defined on  $\text{AdS}_2$  and describes the KK modes including the IIA string excitations. In this section, we investigate the KK spectrum on the  $\text{AdS}_2$  space, especially focusing on the cases of  $k = 1, 2$ .

### 4.1 KK Reduction of bosonic sector

First note that the parameter  $k$  can be absorbed into the definition of  $\rho$  by  $\rho/k \rightarrow \rho$ , and then the period of  $\rho$  becomes  $2\pi/k$ :  $\rho \sim \rho + 2\pi/k$ . For large  $k$ , the circle parametrized by  $\rho$  shrinks and  $\text{AdS}_4 \times \mathbb{S}^7/\mathbb{Z}_k$  reduces to  $\text{AdS}_4 \times \mathbb{CP}^3$ . The conformal field theory on the boundary turns out to be the  $\mathcal{N} = 6$  ABJM theory.

When  $k$  is infinite, the only contribution is the zero modes along  $\rho$  direction. Suppose that  $\eta$  and  $\zeta$  are independent from  $\rho$ , say,  $\eta = \eta_0(\tau, \sigma)$  and  $\zeta = \zeta_0(\tau, \sigma)$ . The bosonic quadratic action (3.18) is reduced to

$$S_B^{(2)} = \frac{1}{2} \frac{2\pi}{k} \int d^2\xi \sqrt{-\det g_0} \left[ g_0^{\hat{i}\hat{j}} (\partial_{\hat{i}} \eta_0 \partial_{\hat{j}} \bar{\eta}_0 + \partial_{\hat{i}} \zeta_0^m \partial_{\hat{j}} \bar{\zeta}_0^m) + 2\eta_0 \bar{\eta}_0 \right], \quad (4.1)$$

where  $\xi^{\hat{i}} = (\tau, \sigma)$  and  $g_{0\hat{i}\hat{j}}$  is the  $\text{AdS}_2$  metric with the unit radius

$$g_{0\hat{i}\hat{j}} = \text{diag}\left(-\frac{1}{\sigma^2}, \frac{1}{\sigma^2}\right). \quad (4.2)$$

The action (4.1) contains one massive complex scalar  $\eta_0$  with  $m^2 = 2$  and three massless complex scalars  $\zeta_0^m$  propagating on the  $\text{AdS}_2$  world-sheet with the metric (4.2).

As for  $k = 1, 2$ , the zero-mode argument would be insufficient because the supersymmetry of the boundary conformal field theory enhances from  $\mathcal{N} = 6$  to  $\mathcal{N} = 8$ . Since the information along the circle is expected to be crucial in this supersymmetry enhancement, we should now take the non-zero KK modes into account.

Let us expand the bosonic fields as

$$\zeta = \sum_{p \in \mathbb{Z}} e^{ip\rho} \zeta_p(\tau, \sigma), \quad \eta = \sum_{q \in \mathbb{Z}} e^{iq\rho} \eta_q(\tau, \sigma). \quad (4.3)$$

Substituting (4.3) into (3.18), we obtain

$$S_B^{(2)} = \frac{2\pi}{2k} \int d^2\xi \sqrt{-\det g_0} \left[ \sum_p \left( g_0^{\hat{i}\hat{j}} \partial_{\hat{i}} \zeta_p^m \partial_{\hat{j}} \bar{\zeta}_p^m + \frac{1}{4} k p (kp + 2) \zeta_p^m \bar{\zeta}_p^m \right) \right]$$

$$+ \sum_q \left( g_0^{\hat{i}\hat{j}} \partial_{\hat{i}} \eta_q \partial_{\hat{j}} \bar{\eta}_q + \frac{1}{4} (k^2 q^2 - 6kq + 8) \eta_q \bar{\eta}_q \right) \Bigg]. \quad (4.4)$$

In this action,  $\zeta_p$  and  $\eta_q$  are described as massive scalars propagating on  $\text{AdS}_2$ .

We first consider the KK spectrum of  $\zeta_p$ . The mass of  $\zeta_p$  is

$$m^2(\zeta_p) = \frac{1}{4} kp(kp + 2). \quad (4.5)$$

The Breitenlohner and Freedman (BF) bound [31] for the  $\text{AdS}_2$  case is given by

$$4m^2 \geq -1.$$

This inequality is satisfied for all  $p$  and is saturated for  $kp = -1$ . Note that  $m^2(\zeta_p)$  is invariant under

$$kp \rightarrow -kp - 2,$$

which implies that  $\zeta_p$  and  $\zeta_{p'}$  have the same mass if  $kp$  and  $kp'$  are related by  $kp' = -kp - 2$ . Interestingly, the pairing of this kind is possible only when

$$p + p' = -\frac{2}{k} \in \mathbb{Z}. \quad (4.6)$$

That is, only  $k = 1$  and  $2$  are possible. For  $k = 1$ , the mode  $\zeta_{-1}$  is not paired and appears alone because  $p' = -p - 2$  for  $p = p' = -1$ .

As for the mode  $\eta_q$ , its mass is

$$m^2(\eta_q) = \frac{1}{4} (kq - 4)(kq - 2). \quad (4.7)$$

The BF bound is satisfied for all the values of  $q$  and is saturated for  $kq = +3$ . The  $m^2(\eta_q)$  is invariant under  $kq \rightarrow -kq + 6$ . Thus there appear two modes with the same mass when  $6/k \in \mathbb{Z}$ , which includes the cases with  $k = 1, 2$ .

The resulting bosonic spectrum for  $k = 1, 2$  is summarized as follows.

$m^2$	$kp$	$kq$	degeneracy
$-1/4$	$-1$	$3$	$8$
$0$	$0, -2$	$2, 4$	$16$
$3/4$	$1, -3$	$1, 5$	$16$
$2$	$2, -4$	$0, 6$	$16$
$15/4$	$3, -5$	$-1, 7$	$16$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

We note that, for the case of  $k = 2$ , only modes with  $kp \in 2\mathbb{Z}$  and  $kq \in 2\mathbb{Z}$  survive. Because  $\zeta_p^m$  give six real scalars and  $\eta_q$  gives two real scalars, we have 8 real scalars for  $m^2 = -1/4$  and 16 real scalars for each of  $m^2 = 0, 3/4, 2, 15/4, \dots$ .



## 4.2 KK Reduction of fermionic sector

Let us turn to the KK spectrum for the fermions.

As in the bosonic case, the fermionic variables are expanded as

$$\vartheta^{\alpha_1 \cdots \alpha_4} = \sum_{r \in \mathbb{Z}} e^{i\rho r} \vartheta_r^{\alpha_1 \cdots \alpha_4}(\tau, \sigma). \quad (4.8)$$

By substituting this expansion into the action (3.36), the KK reduced action is

$$S_F^{(2)} = -8\pi \int d^2\xi \sqrt{-\det g_B^{(0)}} \sum_{\substack{(3.31) \\ r \in \mathbb{Z}}} \left[ \bar{\vartheta}_r \rho^{\hat{i}} \nabla_{\hat{i}} \vartheta_r - \frac{i}{2} (3\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - 2kr) \bar{\vartheta}_r \rho_2 \vartheta_r \right], \quad (4.9)$$

where  $\hat{i} = \tau, \sigma$ . We have suppressed the indices  $\alpha_1, \dots, \alpha_4$  for notational simplicity. The action (4.9) can be rewritten so that it describes the fermions propagating on  $\text{AdS}_2$  with the metric (in unit radius) of (4.2)

$$S_F^{(2)} = -\frac{4\pi}{k} \int d^2\xi \sqrt{-\det g_0} \sum_{\substack{(3.31) \\ r \in \mathbb{Z}}} \left[ \bar{\vartheta}_r \hat{\rho}^{\hat{i}} \nabla_{\hat{i}} \vartheta_r - i\mu_r \bar{\vartheta}_r \rho_2 \vartheta_r \right],$$

$$\mu_r \equiv \frac{1}{4} (3\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - 2kr), \quad (4.10)$$

where we have defined the following quantities

$$\hat{\rho}_{\hat{i}} \equiv \left( \frac{1}{\sigma} \rho_0, \frac{1}{\sigma} \rho_1 \right), \quad \hat{\rho}^{\hat{i}} \equiv (-\sigma \rho_0, \sigma \rho_1). \quad (4.11)$$

Here we note that the covariant derivative in (4.10) may be regarded as the usual derivative:  $\nabla_{\hat{i}} = (\partial_{\tau}, \partial_{\sigma})$ . This is seen easily from (3.22). The term including  $\bar{\Gamma}^{\tau} \Gamma_{03}$  vanishes because  $C\Gamma^0\Gamma_{03} = C\Gamma_3$  is symmetric. And so we may examine (4.10) with the replacement  $\nabla_{\hat{i}} = (\partial_{\tau}, \partial_{\sigma})$  below.

We are interested in the fermion mass spectrum. To read off the fermion mass spectrum, it is convenient to decompose the fermionic variables into the components and scale them like

$$\vartheta_r^{\alpha_1 \cdots \alpha_4} = \begin{pmatrix} \phi_r^{(+)\alpha_1 \cdots \alpha_4} \\ \phi_r^{(-)\alpha_1 \cdots \alpha_4} \end{pmatrix}. \quad (4.12)$$

The equation of motion obtained from (4.10) is rewritten as the coupled system of the first order differential equations

$$\begin{aligned} \sigma(-\partial_{\tau} + \partial_{\sigma}) \phi_r^{(-)} - i\mu_r \phi_r^{(+)} &= 0, \\ \sigma(+\partial_{\tau} + \partial_{\sigma}) \phi_r^{(+)} + i\mu_r \phi_r^{(-)} &= 0. \end{aligned} \quad (4.13)$$

Let us multiply  $\sigma(\partial_\tau + \partial_\sigma)$  to the first equation (and  $\sigma(-\partial_\tau + \partial_\sigma)$  to the second equation, similarly) and introduce  $\phi_r^{(1,2)}$  defined by

$$\phi_r^{(1)} \equiv \phi_r^{(-)} + i\phi_r^{(+)}, \quad \phi_r^{(2)} \equiv \phi_r^{(-)} - i\phi_r^{(+)} . \quad (4.14)$$

Note that  $\phi_r^{(1)}$  is not the complex conjugate of  $\phi_r^{(2)}$  because  $\phi_r^{(+)}$  and  $\phi_r^{(-)}$  are complex variables. As a result,  $\phi_r^{(1,2)}$  should satisfy

$$\begin{aligned} g_0^{ij} \nabla_i \nabla_j \phi_r^{(1)} - \mu_r(\mu_r - 1)\phi_r^{(1)} &= 0, \\ g_0^{ij} \nabla_i \nabla_j \phi_r^{(2)} - \mu_r(\mu_r + 1)\phi_r^{(2)} &= 0, \end{aligned} \quad (4.15)$$

and thus we see that a one two-component complex spinor  $\vartheta_r^{\alpha_1 \cdots \alpha_4}$  represents two massive two-dimensional fermions with masses  $m_F^2 = \mu_r(\mu_r - 1)$  and  $\mu_r(\mu_r + 1)$ , or equivalently

$$m_F^2 = \mu(\mu - 1), \quad \mu = \pm\mu_r . \quad (4.16)$$

By using this mass formula (4.16), let us see the mass spectrum for each of the components  $\vartheta_r^{\alpha_1 \cdots \alpha_4}$ . For  $\vartheta_r^{-+++}$ , since  $\mu_r = -\frac{1}{2}kr$ , the mass spectrum is given by

$kr$	$\mu_r$	$m_F^2(\mu = \mu_r)$	$m_F^2(\mu = -\mu_r)$
-3	3/2	3/4	15/4
-2	1	0	2
-1	1/2	-1/4	3/4
0	0	0	0
1	-1/2	3/4	-1/4
2	-1	2	0
3	-3/2	15/4	3/4

while similarly, for  $\vartheta_r^{+++-}$ ,  $\vartheta_r^{+-++}$  and  $\vartheta_r^{+-+-}$ , since  $\mu_r = \frac{1}{2}(2 - kr)$ ,

$kr$	$\mu_r$	$m_F^2(\mu = \mu_r)$	$m_F^2(\mu = -\mu_r)$
0	1	0	2
1	1/2	-1/4	3/4
2	0	0	0
3	-1/2	3/4	-1/4
4	-1	2	0
5	-3/2	15/4	3/4
6	-2	6	2

Note that the modes with  $kr \in 2\mathbb{Z}$  only survive for the  $k = 2$  case.

Thus the spectrum of the fermionic fluctuations is summarized as follows:

$m_F^2$	$\mu$	degeneracy
$-1/4$	$1/2$	8
0	0	8
	1	8
$3/4$	$-1/2$	8
	$3/2$	8
2	-1	8
	2	8
$\vdots$	$\vdots$	$\vdots$

Since  $\theta_r^{\alpha_1 \dots \alpha_4}$  represents a pair of two-dimensional fermionic modes, there are 8 modes for  $m^2 = -\frac{1}{4}$  and 16 modes for each of  $m^2 = 0, \frac{3}{4}, 2, \frac{15}{4}, \dots$ . The fermionic result nicely agrees with the bosonic one with respect to the value of mass squared and the degeneracy, as it should be.

### 4.3 $\mathcal{N} = 1$ scalar supermultiplets on $\text{AdS}_2$

An  $\mathcal{N} = 1$  scalar supermultiplet on  $\text{AdS}_2$  is composed of a boson with mass  $m_B$  and a fermion with “mass”  $\hat{m}_F$  (which is the coefficient of the fermion mass term and is different from  $m_F$  above), where  $m_B$  and  $\hat{m}_F$  are given by, respectively, (see for example [32])

$$m_B^2 = \mu(\mu - 1), \quad \hat{m}_F = \mu. \quad (4.17)$$

Now we can see that the obtained fluctuations form the  $\mathcal{N} = 1$  scalar supermultiplets. In fact, we can identify  $\mu$  for each of the scalars as follows:

$m_B^2$	$\mu^\#$
$-\frac{1}{4}$	$(1/2)^8$
0	$0^8, 1^8$
$\frac{3}{4}$	$(3/2)^8, (-1/2)^8$
2	$2^8, (-1)^8$
$\frac{15}{4}$	$(5/2)^8, (-3/2)^8$
6	$3^8, (-2)^8$
$\vdots$	$\vdots$

Here  $\#$  denotes the degeneracy. It follows that the fluctuations form the  $\mathcal{N} = 1$  scalar supermultiplets: 8 supermultiplets for each  $\mu \in \mathbb{Z}/2$ , for  $k = 1, 2$ .

As an important remark, they are invariant under  $SO(8)$ . This may suggest  $\mathcal{N} = 8$  supersymmetry of the fluctuations. In fact, it is known that five supermultiplets with  $\mu = 1$  and three supermultiplets with  $\mu = -1$  form an  $\mathcal{N} = 8$  supermultiplet on  $AdS_2$ . It is worth finding out a tower of  $\mathcal{N} = 8$  supermultiplets in our results.

## 5 Fluctuations near the boundary

The next task is to discuss the boundary behavior of the fluctuations. Our purpose here is to identify the non-normalizable modes, which are relevant to the operator insertions into the Wilson loop operator in the field-theory side. The Wilson line we are concerned about is expected to be 1/2 BPS in the  $\mathcal{N} = 8$  Chern-Simons-matter theory, which is the ABJM theory with enhanced  $\mathcal{N} = 8$  supersymmetry.

From the consistency with the equation of motion, we can read off the behavior of a massive scalar field  $\Phi$  near the boundary. When assuming the behavior

$$\Phi \rightarrow \sigma^\lambda \hat{\Phi}(\tau) \quad \text{as } \sigma \rightarrow 0, \quad (5.1)$$

the equation of motion is reduced to

$$-\sigma^{\lambda+2} \partial_\tau^2 \hat{\Phi} + \sigma^\lambda [\lambda(\lambda+1) - m^2] \hat{\Phi} = 0. \quad (5.2)$$

The consistency of (5.2) requires the two conditions:

$$\begin{aligned} \text{(i)} \quad & \lambda + 2 > 0 \quad \text{or} \quad \partial_\tau^2 \hat{\Phi} = 0, \\ \text{(ii)} \quad & \lambda > 0 \quad \text{or} \quad \lambda = \frac{1}{2}(1 \pm \sqrt{1 + 4m^2}) \equiv \lambda_\pm. \end{aligned}$$

These conditions are satisfied for the modes with  $\lambda > 0$  but such modes do not reach the boundary. Similarly, since  $\lambda_+ > 0$ , the mode with  $\lambda = \lambda_+$  does not reach the boundary. So we consider the modes with  $\lambda = \lambda_-$ . The relation between  $\lambda_-$  and  $m^2$  is listed for some values below.

$m^2$	$-1/4$	$0$	$3/4$	$2$	$15/4$	$\dots$
$\lambda_-$	$1/2$	$0$	$-1/2$	$-1$	$-3/2$	$\dots$

As for  $\zeta_p$ , the modes with  $m^2 = -1/4$  cannot reach the boundary because

$$\zeta_p \rightarrow \sigma^{1/2} \hat{\zeta}_p(\tau),$$

which vanishes as  $\sigma \rightarrow 0$ . The modes with  $m^2 \geq 3/4$  diverge as approaching the boundary and cannot be regarded as small fluctuations. Thus the semiclassical approximation is not reliable

any more. The modes with  $m^2 = 0$  correspond to the fluctuations that can reach the boundary consistently with the approximation.

For the case of  $\eta_q$ , it is related to the fluctuation  $\chi_q$  through the field redefinition (3.17). So the corresponding fluctuation  $\chi_q$  behaves as

$$\chi_q = 2\sigma\eta_q \rightarrow \sigma^{\lambda_-+1}\hat{\eta}_q.$$

The modes with  $m^2 \leq 3/4$  cannot reach the boundary and those with  $m^2 \geq 15/4$  diverge. The modes with  $m^2 = 2$  represent the fluctuations that can reach the boundary.

From (4.12) and (4.14), the fermionic modes  $\phi^{(1,2)}$  are related to  $\vartheta$  as

$$\vartheta_r = \begin{pmatrix} \frac{1}{2i}(\phi_r^{(1)} - \phi_r^{(2)}) \\ \frac{1}{2}(\phi_r^{(1)} + \phi_r^{(2)}) \end{pmatrix}.$$

To see the behavior of  $\vartheta$  near the boundary, let us examine that of  $\phi^{(1,2)}$ . Noting that  $\lambda_- = \frac{1}{2}(1 - |2\mu - 1|)$  with  $\mu = \mu_r$  for  $\phi_r^{(1)}$  and  $\mu = -\mu_r$  for  $\phi_r^{(2)}$ , we obtain

$$\phi^{(1)} \rightarrow \sigma^{\frac{1}{2}(1-|2\mu_r-1|)}\hat{\phi}^{(1)}(\tau), \quad \phi^{(2)} \rightarrow \sigma^{\frac{1}{2}(1-|2\mu_r+1|)}\hat{\phi}^{(2)}(\tau).$$

It is straightforward to see that  $\phi^{(1)}$  remains small (or zero) when  $0 \leq \mu_r \leq 1$ , while  $\phi^{(2)}$  does when  $-1 \leq \mu_r \leq 0$ . The equality is satisfied for the mode which remains small and non-zero at the boundary. Namely,  $\phi^{(1)}$  for  $\mu_r = 0, 1$  and  $\phi^{(2)}$  for  $\mu_r = -1, 0$  are the modes reaching the boundary. These are massless  $m_F^2 = 0$ . The fermion with  $\mu_r = 0$  behaves near the boundary as

$$\vartheta \rightarrow \begin{pmatrix} \frac{1}{2i}(\hat{\phi}^{(1)} - \hat{\phi}^{(2)}) \\ \frac{1}{2}(\hat{\phi}^{(1)} + \hat{\phi}^{(1)}) \end{pmatrix}.$$

On the other hand, for the fermion with  $\mu_r = 1$ ,  $\phi^{(1)}$  remains small but  $\phi^{(2)}$  goes beyond semiclassical approximation. Similarly for a fermion with  $\mu_r = -1$ ,  $\phi^{(2)}$  remains small but  $\phi^{(1)}$  goes beyond semiclassical approximation. As a result, the fermions reaching the boundary are the following: the fermion with  $\mu_r = 0$ , a half of the fermion with  $\mu_r = 1$  and a half of the fermion with  $\mu_r = -1$ .

In summary, the following modes can reach the boundary:

- $\zeta_p^m$  ( $m = 1, 2, 3$ ) with  $kp = 0, -2$
- $\eta_q$  with  $kq = 0, +6$
- $\vartheta_r^{-+++}$  with  $kr = 0$

- $\phi_r^{(1)-+++}$  with  $kr = -2$   
 $\phi_r^{(2)-+++}$  with  $kr = 2$
- $\vartheta_r^{++++}$ ,  $\vartheta_r^{++--}$  and  $\vartheta_r^{+-+-}$  with  $kr = 2$
- $\phi_r^{(1)+++-}$ ,  $\phi_r^{(1)++-+}$  and  $\phi_r^{(1)+--+}$  with  $kr = 0$   
 $\phi_r^{(2)+++-}$ ,  $\phi_r^{(2)++-+}$  and  $\phi_r^{(2)+--+}$  with  $kr = 4$

The consistency condition (i) is satisfied for all these modes. They are 16 bosonic fluctuations and 16 fermionic ones, which can be regarded as to deform the Wilson line in the boundary gauge theory. In particular, 8 bosons and 5 fermions are contained as zero modes.

Among the modes that can reach the boundary, the zero modes may be problematic when we consider the case of  $k > 2$ . In this case, only the zero modes survive in type IIA limit, and the 5 fermionic zero modes seem weird. Probably, the reason would be related to the fixing condition for the kappa symmetry and to the fact that 8 supersymmetries are inevitably broken in moving to type IIA description.

Hence there is subtlety of how to pick up the fermionic degrees of freedom that should be removed by using the  $\kappa$  symmetry. We have taken a possible fixing condition for the  $\kappa$  symmetry, but it does not seem to be compatible with the one in type IIA Green-Schwarz string discussed in [33, 34]. A comment on this point is given in the next section. There may be another appropriate gauge fixing procedure which enables the fermion zero modes to survive all the way to the boundary. An optimistic possibility is that our result may be correct in some way. In order to explain the origin of the fermionic degrees of freedom contained in the 1/2 BPS Wilson loops, there may be a non-trivial mechanism concerning the fermion sector as the theory flows from M-theory region to type IIA string theory.

As for the number of degrees of freedom in each of bosonic and fermionic fluctuations, our result 16 seems twice the expected number and one may feel curious. At present, we do not have clear understanding for the redundant degrees of freedom. Those may agree with the results on 1/2 BPS Wilson loops recently discussed in [19], where the Wilson loops are discussed as the Higgsing in the ABJM model and then there appear the two multiplets corresponding to  $(\mathbf{N} - \mathbf{1}, \mathbf{1})$  and  $(\mathbf{1}, \mathbf{N} - \mathbf{1})$  representations after the Higgsing. Since this property may persist in the  $\mathcal{N} = 8$  case, the multiplets may correspond to our results though we do not have any confirmation with the exact matching of the degrees of freedom. As another possibility, we may argue that those would probably correspond to the non-ABJM fields discussed in [35] because we have started from the eleven-dimensional  $\mathcal{N} = 8$  setup with  $k = 1, 2$ .

## 6 Summary and Discussion

In this paper we have considered semiclassical fluctuations around a single M2-brane configuration on  $\text{AdS}_4 \times \text{S}^7/\mathbb{Z}_k$ . The configuration is static, 1/2 BPS and its shape is  $\text{AdS}_2 \times \text{S}^1$ . We have investigated the Kaluza-Klein reduction on  $\text{S}^1$  and shown that the resulting fluctuations form an infinite set of  $\mathcal{N} = 1$  supermultiplets on  $\text{AdS}_2$  for  $k = 1, 2$ . The  $\text{SO}(8)$  invariance of the spectrum may suggest the  $\mathcal{N} = 8$  supersymmetry on  $\text{AdS}_2$ .

We also have discussed the behavior of the fluctuations near the boundary of  $\text{AdS}_2$ . As a result, it has been shown that 16 bosonic fluctuations and 16 fermionic ones can reach the boundary without spoiling the semiclassical approximation. It seems twice the expectation but we argue that the redundant degrees of freedom should correspond to the non-ABJM fields because we have started from the eleven-dimensional  $\mathcal{N} = 8$  setup with  $k = 1, 2$ .

We have noted a subtlety for the fixing condition of the kappa symmetry. In other words, this problem is translated to the choice of classical solution in a sense that our  $\kappa$ -symmetry fixing condition (3.21) depends on  $\Gamma$  whose form is fixed once we choose a classical configuration. In the present case,  $\Gamma$  in (2.25) is fixed by choosing the classical configuration (3.1). After the  $\kappa$ -gauge fixing we are left with 8 spinors in (3.31) and (3.32). Under the breaking  $\text{SO}(8) \rightarrow \text{SU}(4) \times \text{U}(1)$ , the  $8_s$  representation decomposes into  $1_2 + 1_{-2} + 6_0$ . Then  $1_2$  and  $1_{-2}$  correspond to  $\vartheta^{-+++}$  and  $\vartheta^{+---}$ , respectively, and  $6_0$  consists of  $\vartheta^{-+-}$ ,  $\vartheta^{--+}$ ,  $\vartheta^{---+}$ ,  $\vartheta^{+-++}$ ,  $\vartheta^{++-+}$ ,  $\vartheta^{+++}$ <sup>6</sup>. If we start from the partially  $\kappa$ -gauge fixed type IIA string action in  $\text{AdS}_4 \times \mathbb{CP}^3$  [33, 34],  $\vartheta^{-+++}$  and  $\vartheta^{+---}$  are absent from the outset. On the other hand, our result contains these spinors (even for zero modes). Hence our result cannot be derived from the IIA perspective. It is interesting to examine different M2-brane configurations from the one we considered here and to investigate the relation to type IIA theory.

Our aim here was to confirm the correspondence between (dimensionally reduced) M2-brane world-volume and Wilson loop from the analysis of the fluctuations in the case of  $\text{AdS}_4/\text{CFT}_3$  as in  $\text{AdS}_5/\text{CFT}_4$ . For this purpose the fermion zero-modes in our result look weird. This may be because the  $\kappa$ -gauge fixing condition forced by the choice of the classical solution is not compatible with the IIA string setup. We hope that there may be a nice interpretation to support our result. In any case, at the present stage, it is hard to answer whether the correspondence should hold or not. We need much effort to give a definite answer and it remains as a future problem.

Another way is to consider semiclassical fluctuations around the  $\text{AdS}_2$  solution by starting

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<sup>6</sup> The  $\text{U}(1)$  charges  $(s_1, s_2, s_3, s_4)$  in [15] correspond to  $(\gamma_9 \alpha_1, \alpha_2, \alpha_3, \alpha_4)$  with  $\gamma_9 = -1$  in our notation. Our spinors are complex subject to the reality condition (3.35) and we may regard (3.31) as independent ones.

directly from type IIA string theory on  $\text{AdS}_4 \times \mathbb{CP}^3$  rather than the M2-brane theory. It would be interesting to consider the interpretation of the fluctuations as a small deformation of the 1/2 BPS Wilson line [18] or the 1/6 BPS Wilson line [15–17]. We hope that we could report some results in this direction in the near future [36].

There are some other open problems. One of them is to compute the semiclassical partition function around the static M2-brane solution as in the case of  $\text{AdS}_5/\text{CFT}_4$  [21]. Another one is to consider a Wilson loop in the  $\mathcal{N} = 8$  three-dimensional CFT like the BLG theory [4, 5]. This has not been clarified at all, and so it would be nice if we could shed light on this issue from our approach.

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